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Quantized controllers distributed over a network: An industrial case study

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***Abstract*—We consider the problem of regulating pressures across large-scale hydraulic networks. We investigate the use of a class of piece-wise constant control laws which take value in a finite number of values and whose transition from one value to another occurs when the measurements cross certain thresholds. We show that these controllers guarantee set-point pressure regulation with an arbitrarily large domain of convergence. The use of this class of control laws is motivated by the need to exchange information among controllers distributed over the network.**

I. Introduction

In a recent work ([2]) we have studied control strategies distributed over a network to meet heat demand in large-scale district heating systems. The problem boils down to regulate to a constant reference value pressure drops across end-users valves in the underlying hydraulic network. We have characterized a large class of hydraulic networks, which include all the networks which are important from the practical point of view, and we have shown that *proportional* and *proportional-integral* control laws which use local measurements only, namely the pressure drops at the end-users, allow to achieve pressure regulation across the entire network from any arbitrarily large set of initial conditions. Preliminary experimental validation of our results have also been obtained ([2]).

In this document, we investigate the use of a class of piece-wise constant control laws which take value in a finite number of values and whose transition from one value to another occurs when the measurements cross certain *thresholds*. These control laws are known as quantized controllers ([5], [6], [7], [1]). We show that quantized controllers guarantee practical pressure regulation in the network with an arbitrarily large domain of convergence. We also examine a special class of quantized controllers, namely *binary* controllers. The motivation to use quantized or binary controllers in the control of hydraulic networks can be explained as follows. The implementation of the control laws requires an exchange of information among the controllers. Since these controllers are remotely located, the transmission of information requires a communication channel. Quantized control laws take value in a finite set, and as such they are compatible with the presence of a finite data-rate channel. Moreover, since quantized control laws take new values only when certain events occur (the crossing of thresholds by measurement signals), they are also compatible with the scenario in which the communication medium is a limited resource and transmission must take place sporadically.

In the next section, we present the industrial case study we are interested in. In Section 3, we discuss the need of an exchange of information among the controllers distributed over the network. Section 4 deals with quantized and binary controllers. It introduces the pressure regulation problem using binary controllers, and presents the main result of the paper. Simulation results are illustrated in Section 5.

II. An industrial case study

The industrial case study we consider is concerned with the fulfillment of heat demands in large-scale district heating systems, a problem for which some results have been presented in the papers [2], [3]. A district heating systems comprises several end-users with different heat demands, and the problem is that of guaranteeing the demands of all the end-users despite of the erratic nature of the demands themselves. After introducing the hydraulic network underlying the district heating system, the problem can be reduced to a pressure regulation problem in a large-scale hydraulic network.

A. Hydraulic networks

An hydraulic network is a connection of two-terminal components such as valves, pipes and pumps, whose constitutive laws put in relation the pressure drop $\Delta h = h_i - h_j$ across the element and the flow q through the element. We briefly recall the constitutive laws of these components referring the reader to [2], [3] for more details.

A *valve* is characterized by the algebraic relation

$$h_i - h_j = \mu(K_v, q)$$

where K_v is the hydraulic resistance of the valve, and μ is a smooth function of its arguments which, for each fixed value of K_v is zero at zero and strictly increasing. The constitutive law of a *pipe* is a dynamic relation of the type

$$J\frac{dq}{dt} = (h_i - h_j) - \lambda(K_p, q)$$

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with J, K_p parameters and λ a function which enjoys the same properties of the function μ. Finally, a (centrifugal) *pump* is a device which delivers the desired pressure difference $h_i - h_j$ no matter what is the flow through it. The constitutive law of the pump is

$$h_i - h_j = -\Delta h_p$$

where Δh_p is a nonnegative function of time which is viewed as a control input.

The value of the parameters K_v, K_p are typically unknown and we shall assume they range over a compact sets of strictly positive values, denoted by $\mathcal{P}$. Similarly, the functions μ, λ are not precisely known, and in fact knowing them is not necessary for the analysis, at least as far as the two properties of smoothness and monotonicity are guaranteed. We will distinguish between end-user valves and the other valves, allowing the former to change the value of the hydraulic resistance in a piece-wise constant fashion, and between the end-user pumps – located in the vicinity of the end-user valves – and the boosting pumps, that is pumps used to fulfill constraints on the relative pressures across the network which the end-user pumps alone – mainly used to meet the demands of the end-users – could not fulfill.

B. Model

To derive a model for these systems, it is convenient and natural to resort to tools in circuit theory ([4]). We will not review all the details here, referring the interested reader to [4], [3]. Rather, we will only recall the few notions which are needed to follow the developments below. In particular, we associate to the hydraulic network a graph $\mathcal{G}$ whose nodes are the terminals of the network's components and whose edges are the components themselves. Then a set of $n \geq 1$ *independent* flow variables (i.e. a set of flow variables whose value can be set independently from all the other flows in the network) are singled out. These independent flows coincide with the flows through the so-called *chords* of the graph ([4], [3]). A fundamental loop is associated to each chord, and along each fundamental loop Kirchhoff's voltage law holds, that is $B\Delta h = \mathbf{0}_{n\times 1}$, where B is called the *fundamental loop matrix*, i.e. a matrix of $-1, 1, 0$ whose value depends on the topology of the circuit, and Δh is the vector of all the pressure drops across the components of the network.

We are now ready to present the class of hydraulic networks which are important for our case study. This class satisfies the following two assumptions:

Assumption 1: Each user valve is in series with a pipe and a pump, see Fig. 1. Moreover, each chord in $\mathcal{G}$ corresponds to a pipe in series with a user valve.

Assumption 2: There exists one and only one component called the *heat source*. It corresponds to a valve[1] of the network, and it lies in all the fundamental loops.

Remark. The assumption appears very mild, since in district heating systems it is typical to have only one common heat source which has to provide hot water to all the end-users. Hence, it is natural to expect the heat source to lie in all the fundamental loops of the network.

[1]The valve models the pressure losses in the secondary side of the heat exchanger of the heat source.

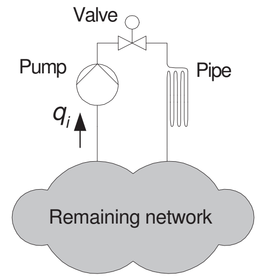

Fig. 1. The series connection associated with each end-user.

The following result holds ([3]):

Proposition 1: Any hydraulic network satisfying Assumption 1 admits the model

$$\begin{array}{rcl} J\dot{q} &=& f(K_p, K_v, B^T q) + u \\ y_i &=& \mu_i(k_{vi}, q_i)\ ,\ i = 1, 2, \ldots, n \end{array} \tag{1}$$

with $q \in \mathbb{R}^n$ the vector of independent flows, $u \in \mathbb{R}^n$ a vector of n independent inputs, y_i the measured pressure drop across the ith end-user valve, $J = J^T > 0$ an $n \times n$ matrix, K_p, K_v vectors of parameters, $f(K_p, K_v, B^T q)$ a smooth vector field, $\mu_i(k_{vi}, q_i)$ the constitutive law of the ith end-user valve.

The model has some nice features among which we recall the following, which states that if all the flows in the network have positive sign and there is no input action, then all the entries of the flow velocity vector $J\dot{q}$ are strictly negative. Namely we have ([3]):

Lemma 1: Under Assumptions 1 and 2, $q \in \mathbb{R}^n_+$ [2] implies $-f(K_p, K_v, q) \in \mathbb{R}^n_+$.

The input vector u deserves a few comments too. As a matter of fact, it can be shown ([3]) that $u = B\Delta h_p$, with B, the fundamental matrix recalled above, and Δh_p, the vector of pump pressures, taking respectively the form

$$B = \begin{pmatrix} I & I & F' \end{pmatrix}\ ,\ \Delta h_p = \begin{pmatrix} 0 \\ \Delta h_p^e \\ \Delta h_p^b \end{pmatrix}\ ,$$

with Δh_p^e, Δh_p^b the vectors of pressures delivered by the end-user pumps and, respectively, the boosting pumps. The sub-matrix F' turns out to have all *non-negative* entries as a consequence of Assumption 2.

Example. A "reduced-size" district heating system with three pumps and three heat exchangers, and supplying two apartment buildings was considered in [2] (see Fig. 2). The corresponding hydraulic network is depicted in Fig. 3. The two end-users are represented by the valves c_6, c_{13}, the two end-user pumps coincide with the components c_5, c_{12}, and the boosting pump is c_1. Applying the techniques sketched

[2]$\mathbb{R}^n_+$ denotes the positive orthant of $\mathbb{R}^n$, i.e. the set $\{q \in \mathbb{R}^n : q_i > 0\,,\, i = 1, \ldots, n\}$.

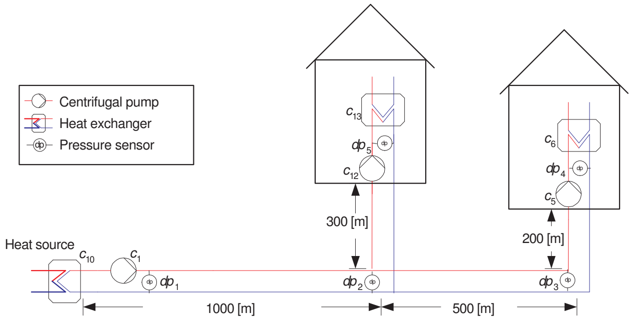

Fig. 2. A sketch of a small district heating system.

above, that is applying Kirchhoff's voltage law to the two fundamental circuits which in this case can be easily identified by inspection, it is possible to derive the following equations:

$$\begin{aligned}
&\begin{pmatrix} \sum_{i\in\mathcal{P}_1} J_i & \sum_{i\in\mathcal{P}_{12}} J_i \\ \sum_{i\in\mathcal{P}_{12}} J_i & \sum_{i\in\mathcal{P}_2} J_i \end{pmatrix}\begin{pmatrix}\dot{q}_1\\ \dot{q}_2\end{pmatrix} \\
&= \begin{pmatrix} -(\sum_{i\in\mathcal{P}_1\setminus\mathcal{P}_{12}} K_{pi})|q_1|q_1 - (\sum_{i\in\mathcal{P}_{12}} K_{pi})|q_1+q_2|\cdot \\ -(\sum_{i\in\mathcal{P}_2\setminus\mathcal{P}_{12}} K_{pi})|q_2|q_2 - (\sum_{i\in\mathcal{P}_{12}} K_{pi})|q_1+q_2|\cdot \end{pmatrix. \\
&\qquad \begin{matrix} \cdot(q_1+q_2) - K_{v6}|q_1|q_1 - K_{v10}|q_1+q_2|(q_1+q_2) \\ \cdot(q_1+q_2) - K_{v13}|q_2|q_2 - K_{v10}|q_1+q_2|(q_1+q_2) \end{matrix}\Bigg) \\
&+\begin{pmatrix}\Delta h_{c_5} + \Delta h_{c_1} \\ \Delta h_{c_{12}} + \Delta h_{c_1}\end{pmatrix}
\end{aligned}$$

where $\mathcal{P}_1 = \{2,3,4,7,8,9\}, \mathcal{P}_{12} = \{2,9\}, \mathcal{P}_2 = \{2,9,11,14\}$, and where we have set $\mu(K_v,q) = K_v|q|q, \lambda(K_p,q) = K_p|q|q$ ([2]). In the present case:

$$u = \begin{pmatrix}\Delta h_{c_5} + \Delta h_{c_1} \\ \Delta h_{c_{12}} + \Delta h_{c_1}\end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1\end{pmatrix}\begin{pmatrix}\Delta h_{c_5} \\ \Delta h_{c_{12}} \\ \Delta h_{c_1}\end{pmatrix}. \tag{2}$$

◁

III. Communication topology

The control problem for the system introduced in the previous section is to regulate the pressure drops y_i at a desired set point r_i by appropriate design of the control input u_i. The

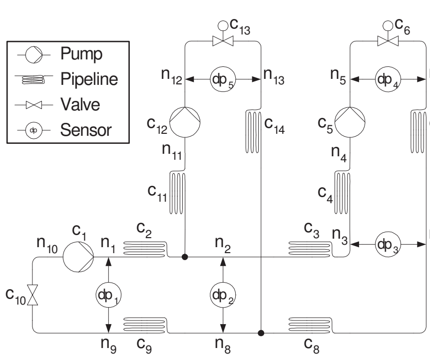

Fig. 3. The hydraulic network diagram.

control vector u is related to the actual control inputs Δh_p^e, Δh_p^b through the identity $u = B\Delta h_p$ or, what is the same,

$$\begin{aligned}
u &= \begin{pmatrix} I & I & F' \end{pmatrix}\begin{pmatrix} 0 \\ \Delta h_p^e \\ \Delta h_p^b \end{pmatrix} \\
&= \begin{pmatrix} I & F' \end{pmatrix}\begin{pmatrix} \Delta h_p^e \\ \Delta h_p^b \end{pmatrix} \doteq \overline{B}\cdot\overline{\Delta h_p}\,.
\end{aligned}$$

Let ℓ be the number of pumps (both end-user and boosting pumps) present in the network, i.e. let $\Delta h_p \in \mathbb{R}_+^\ell$. For practical implementation, it is important to find the expressions of $\overline{\Delta h_p}$ as functions of u. Since $\overline{B}$ is a full-row-rank matrix, the pseudo-inverse $\overline{B}^\dagger = \overline{B}^T(\overline{B}\,\overline{B}^T)^{-1}$ is such that $\overline{\Delta h_p} = \overline{B}^\dagger u$ is a solution of $u = B\Delta h_p$. Yet, there might be other ways to map u to Δh_p, more convenient from the point of view of the application. For instance, in the example examined previously, it is possible to map u to Δh_p in such a way that, if u is positive component-wise, so is the vector Δh_p, thus fulfilling the positivity constraint imposed by the use of centrifugal pumps.

Example. (Cont'd) The relation $u = B\Delta h_p$ in the example above is described in (2). It is easy to find the expressions of $\Delta h_{c_5}, \Delta h_{c_{12}}, \Delta h_{c_1}$ as functions of u_1, u_2 in such a way that $u = B\Delta h_p$ holds. For instance, one can define

$$\Delta h_{c_5} = \frac{2}{3}u_1 - \frac{1}{3}u_2\,,\ \Delta h_{c_{12}} = -\frac{1}{3}u_1 + \frac{2}{3}u_2\,,$$
$$\Delta h_{c_1} = \frac{1}{3}u_1 + \frac{1}{3}u_2\,.$$

However, this choice does not guarantee that $\Delta h_{c_5}, \Delta h_{c_{12}}, \Delta h_{c_1}$ are all positive if so are u_1, u_2. On the other hand, one can verify that this property is guaranteed if we define:

$$\begin{aligned}
\Delta h_{c1} &= \kappa\min\{u_1,u_2\} \\
\Delta h_{c5} &= u_1 - \kappa\min\{u_1,u_2\} \\
\Delta h_{c12} &= u_2 - \kappa\min\{u_1,u_2\}\,,
\end{aligned} \tag{3}$$

with $0 < \kappa < 1$. ◁

In what follows, we shall assume that for each component i of $\overline{\Delta h_p}$ there exists a continuous function δ_i such that

$$\overline{\Delta h_{pi}} = \delta_i(u_{j_{i1}}, \ldots, u_{j_{in_i}})\,,$$

with $j_{i1}, \ldots, j_{in_i}$ a subset of the indices $\{1,2,\ldots,n\}$ such that $\overline{\Delta h_{pi}}$ depends non trivially on $u_{j_{i1}}, \ldots, u_{j_{in_i}}$. These functions δ_i also define the topology of the communications which must take place for the actual implementation of the control laws. Indeed, since u_i is the control law computed by the local controller located at the end-user i, the pump which delivers the pressure $\overline{\Delta h_{pi}}$ needs to know all the control laws u_i which appear explicitly in $\delta_i(u_{j_{i1}}, \ldots, u_{j_{in_i}})$. The information must be transmitted from local controllers located at the end-users $j_{i1}, \ldots, j_{in_i}$ to the pump i.

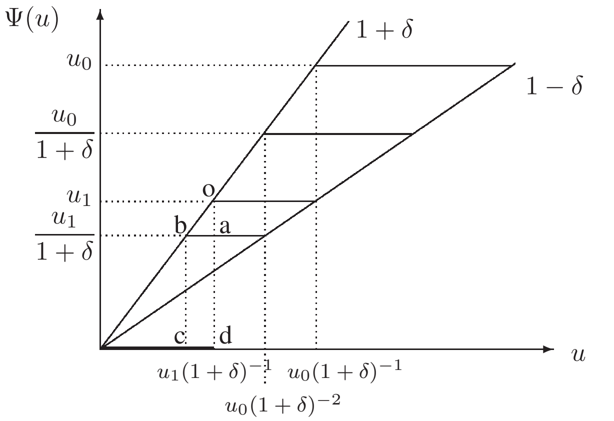

Fig. 4. The multi-valued map $\Psi_m(u)$ for $u > 0$, and with $j = 1$.

IV. Quantized controllers and the pressure regulation problem

We have seen above that, given a class of control laws u_i, their actual implementation requires information transmission among the end-users. Since the end-users are located far away from each other, one can expect that in future implementations of these control laws, a communication channel will lie among the controllers. Bearing in mind this scenario, we focus here on piece-wise constant control laws which take value in a finite set and whose evolution is determined by the occurrence of certain events in the measurement space. Although we will give the main result in terms of binary controllers, a similar statement can be derived for quantized controllers as well, and hence we present such a class of controllers for the sake of completeness. In what follows, $\epsilon_i = y_i - r_i$ denotes the pressure regulation error, with r_i the set point to which the output y_i must be regulated.

A. Quantized controllers

By quantized control law it is typically meant a piece-wise constant function of time taking values in a finite set. In this paper, following [1], we consider quantized control laws $u_i = \Psi_m(\epsilon_i)$ with Ψ_m the following multi-valued map (the map is depicted in Fig. 4):

$$\Psi_m(u) = \begin{cases} \psi_i & \dfrac{\psi_i}{1+\delta} < u \le \dfrac{\psi_i}{1-\delta}\ , \\ & \qquad\qquad 0 \le i \le j \\ \dfrac{\psi_i}{1+\delta} & \dfrac{\psi_i}{(1+\delta)^2} < u \le \dfrac{\psi_i}{(1+\delta)(1-\delta)}\ , \\ & \qquad\qquad 0 \le i \le j \\ 0 & u \le \dfrac{\psi_j}{1+\delta}\ . \end{cases} \tag{4}$$

In the definition above, j is a positive integer, u_0 is a positive real number, $\delta \in (0,1)$, and $\psi_i = \rho^i \psi_0$ for $i = 1, 2, \ldots, j$ with $\rho = \frac{1-\delta}{1+\delta}$. The parameters j, ψ_0, δ are to be designed. Since the map above is multi-valued, we need to specify the law according to which $\Psi_m(\epsilon_i(t))$ changes its value as $\epsilon_i(t)$ evolves. This law is illustrated by the graph in Fig. 5. The description below is taken verbatim from [1]. At time $t = 0$, we fix $\Psi_m(\epsilon_i(0))$. This value of $\Psi_m(\epsilon_i(0))$ identifies a node of the graph. If the value of $\epsilon_i(0)$ fulfills one of the conditions of the edges leaving the node, then a transition is triggered and the quantizer takes the new value – which is denoted by $\Psi_m(\epsilon_i(0^+))$ – given by the destination node. For $t > 0$, $\Psi_m(\epsilon_i(t))$ remains constant until $\epsilon_i(t)$ triggers a transition of $\Psi_m(\epsilon_i(t))$ to the new value, denoted by $\Psi_m(\epsilon_i(t^+))$, again chosen according to the graph of Fig. 5. We refer to [7], Section 3, for further details on the switching mechanism.

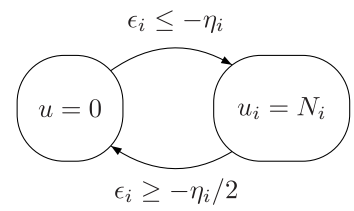

Fig. 6. The hysteretic binary controller.

B. Binary controllers

A binary controller can be described as follows. Let η_i be a positive constant and $u_i = \Psi_{ti}(\epsilon_i)$ the law generated by the binary controller. At the initial time $t = 0$, set the control value as

$$u_i(0) = \begin{cases} 0 & if \quad \epsilon_i(0) > -\eta_i \\ N_i & if \quad \epsilon_i(0) \le -\eta_i\ , \end{cases} \tag{5}$$

with $N_i > 0$ a design parameter. For $t \ge 0$, the control law evolves according to the automaton in Fig. 6. Denote the resulting control law by

$$u_i(t) = \Psi_{ti}(\epsilon_i(t)) = \Psi_{ti}(y_i(t) - r_i(t))\ , \quad i = 1, 2, \ldots, n\ . \tag{6}$$

C. The pressure regulation problem

In this section we introduce the pressure regulation problem in the presence of the communication constraints discussed above. We adopt the controllers (6) which present four main features: (i) They provide a *positive* control action to cope with the constraint that centrifugal pumps, which are the most common actuators found in real hydraulic networks, only provide positive pressures; (ii) They use information which is available *locally* at the end-user, namely the regulation error $y_i - r_i$; (iii) They can be transmitted over a finite-bandwidth transmission channel, since they take a finite number of values; (iv) They are *event-based*, since a new value is taken whenever the regulation error $y_i - r_i$ crosses certain boundaries, and as such more suitable for those cases in which transmission must be taken sporadically.

The problem that these binary proportional controllers can solve is the following. Suppose that the set-point values r_i, $i = 1, 2, \ldots, n$, are known and constant, taking values in the

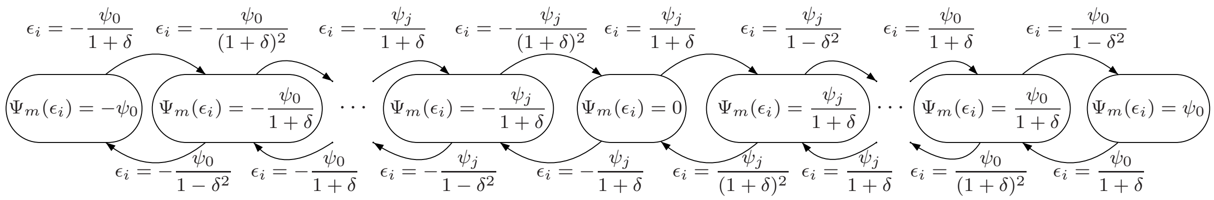

Fig. 5. The graph illustrates how the function $\Psi_m(\epsilon_i)$ takes values depending on ϵ_i. Each edge connects two nodes, and is labeled with the condition (*guard*) which triggers the transition from the starting node to the destination node.

compact set $\mathcal{R} \subset \mathbb{R}^n_+$, namely $\mathcal{R} = \{r \in \mathbb{R}^n \,:\, 0 < r_m \leq r_i \leq r_M \,,\, i = 1, \ldots, n\}$.

Pressure Regulation Problem. Given system (1), a compact set of parameters $\mathcal{P}$, a compact interval of reference values $\mathcal{R}$, a compact set of initial conditions

$$\mathcal{Q} = \{q \in \mathbb{R}^n \,:\, |q_i| \leq q_M \,,\, i = 1, \cdots, n\} \,, \tag{7}$$

and an arbitrarily small positive number γ, find controllers of the form (6), such that, for any $(K_p, K_v) \in \mathcal{P}$ and $r \in \mathcal{R}$, every trajectory $q(t)$ of the closed-loop system (1), (6) with initial condition in $\mathcal{Q}$ is attracted [3] by the set $\{\epsilon \in \mathbb{R}^n \,:\, |\epsilon_i| \leq \gamma \,,\, i = 1, \cdots, n\}$, where $\epsilon_i = y_i - r_i$.

Remark. Finding controllers of the form (6) amounts to finding the parameters N_i, η_i in Ψ_{ti}, for $i = 1, 2, \ldots, n$. $\triangleleft$

A modification of the results of [2] yields the following result:

Proposition 2: For any choice of the parameter $q_M > 0$, of the compact sets $\mathcal{R} \subset \mathbb{R}_+$, and $\mathcal{Q}$ in (7), and for any arbitrarily small positive number γ, there exist parameters N_i, η_i, for $i = 1, 2, \ldots, n$ such that, for any $r \in \mathcal{R}$, any trajectory $q(t)$ of the closed-loop system (1),(6) with initial condition in $\mathcal{Q}$ is attracted by the set $\{\epsilon \in \mathbb{R}^n \,:\, |\epsilon_i| \leq \gamma \,,\, i = 1, 2, \ldots, n\}$, where $\epsilon_i = y_i - r_i$.

Proof: First, it is not difficult to realize that each function $\mu_i(K_{vi}, q_i)$, which is the function which returns the pressure drop across the end-user valve, admits an inverse μ_i^{-1} which satisfies $q_i = \mu_i^{-1}(K_{vi}, \mu_i(K_{vi}, q_i))$ for all K_{vi}. Then define the error coordinates $e = q - \mu^{-1}(K_v, r)$, where

$$\mu^{-1}(K_v, r) = \begin{pmatrix} \mu_1^{-1}(K_{v1}, r_1) & \ldots & \mu_n^{-1}(K_{vn}, r_n) \end{pmatrix}^T \,.$$

It is possible to verify (see [3]) that $\epsilon_i(e_i, K_{vi}, r_i) = \mu_i(K_{vi}, e_i + \mu_i^{-1}(K_{vi}, r_i)) - r_i$ is a monotonically increasing function of e_i, $\epsilon_i(0, K_{vi}, r_i) = 0$ and

$$\epsilon_i(e_i, K_{vi}, r_i) e_i > 0 \quad \text{for all } e_i \neq 0 \,. \tag{8}$$

The system (1) in the e-coordinates rewrites as

$$J\dot{e} = f(K_p, K_v, B^T q)\big|_{q = e + \mu^{-1}(K_p, K_v, r)} + u \,. \tag{9}$$

To prove the result, we use Lyapunov arguments. We choose as Lyapunov function $V(e) = e^T J e$, with J the symmetric positive definite matrix in (1). In particular, let Γ_σ, Γ_ρ two levels sets which, respectively, contain the set of initial conditions

$$\mathcal{E} = \{e \in \mathbb{R}^n : e = q_f - \mu^{-1}(K_v, r), q_f \in \mathcal{Q}, r \in \mathcal{R}, K_v \in \mathcal{P}\} \,,$$

[3]We mean that a trajectory is *attracted* by a set S if it is defined for all $t \geq 0$, and it belongs to S for all $t \geq T$, with $T > 0$ a finite time.

with $\mathcal{Q}$ as in (7), and include the target set $\{e \in \mathbb{R}^n \,:\, |\epsilon_i(e_i, K_{vi}, r_i)| \leq \gamma \,,\, i = 1, \cdots, n\}$. Moreover, let γ' be such that $\{e \in \mathbb{R}^n \,:\, |e_i| \leq \gamma' \,,\, i = 1, \cdots, n\} \subseteq \Gamma_\rho$. Finally define, $S = \Gamma_\sigma \setminus \mathring{\Gamma}_\rho$, with $\mathring{\Gamma}_\rho$ the interior of Γ. Now observe that the Lyapunov function $V(e(t))$ computed along the solutions of the closed-loop system (1),(6) is differentiable for all times t except for those at which the controller (6) switches. At these times, $V(e(t))$ is continuous. As a consequence, if we can prove that, for all the times t for which $V(e(t))$ is differentiable and $e(t) \in S$, $\dot{V}(e(t)) < 0$, then we can guarantee that for a certain finite time T, $V(e(t)) \leq \rho$ for all $t \geq T$. To prove $\dot{V}(e(t)) < 0$ for (almost) all t for which $e(t) \in S$, we follow the lines of [2]. In particular, we single out a number of regions in S for the system and analyze the sign of $\dot{V}(e(t))$. First, let $M > 0$ be such that $e^T f(K_p, K_v, B^T q)\big|_{q = e + \mu^{-1}(K_p, K_v, r)} < M$ for all $e \in S$, all $K_p, K_v \in \mathcal{P}$, and all $r \in \mathcal{R}$. Then consider the following sub-regions of S.

Region 1 ($\mathcal{R}_1 = \{e \in S \,:\, e_i \leq 0, i = 1, \ldots, n\}$). The expression of $\dot{V}(e(t))$ along the solutions of the closed-loop system is (the dependence on t is dropped)

$$\begin{aligned} \dot{V}(e) \;\; = \;\; & 2e^T f(K_p, K_v, B^T q)\big|_{q = e + \mu^{-1}(K_p, K_v, r)} + \\ & \sum_{i=1}^{n} e_i \Psi_{ti}(\epsilon_i(e_i, K_{vi}, r_i)) \,. \end{aligned} \tag{10}$$

By the definition of γ', any point e in $\mathcal{R}_1$ is such that $|e_{j(e)}| \geq \gamma'$ for at least an index $j(e) \in \{1, \cdots, n\}$. In (6), choose η_i in such a way that $-\eta_i \geq \epsilon_i(-\gamma', K_{vi}, r_i)$. Therefore

$$\sum_{i=1}^{n} e_i \Psi_{ti}(\epsilon_i(e_i, K_{vi}, r_i)) \leq -N_{j(e)} \gamma' \,.$$

Then, choosing N_{i1} in such a way that $M - N_{i1}\gamma' < 0$ for all $i \in \{1, \cdots, n\}$, we have $\dot{V}(e) < 0$ for all $e \in \mathcal{R}_1$, for any $N_i \geq N_{i1}$.

Region 2 ($\mathcal{R}_2 = \{e \in S \,:\, e_i \geq 0 \,,\, i = 1, \ldots, n\}$). Due to the definition of the controller, in this region $u = 0$. By Lemma 1, it holds that $\dot{V}(e) < 0$ for all $e \in \mathcal{R}_2$.

Region 3 $\mathcal{R}_3 = S \setminus (\mathcal{R}_1 \cup \mathcal{R}_2)$. The set $\mathcal{R}_3$ is further partitioned as follows. Observe first that there exists $2^n - 2$ non-void intersection $\mathcal{R}_3$ with the orthants of $\mathbb{R}^n$. Call these intersections $\mathcal{R}_{3\ell}$, with $\ell = 1, \ldots, 2^n - 2$, and consider the partition $\mathcal{R}_3 = \cup_{\ell=1}^{2^n-2} \mathcal{R}_{3\ell}$. Associated with each sub-region $\mathcal{R}_{3\ell}$ there exists a unique set of indices $\mathcal{L}_\ell \subset \{1, \ldots, n\}$ such that $e \in \mathcal{R}_{3\ell}$ if and only if $e_i \leq 0$ for each $i \in \mathcal{L}_\ell$ and $e_i \geq 0$ for each $i \in \bar{\mathcal{L}}_\ell$, with $\bar{\mathcal{L}}_\ell = \{1, \ldots, n\} \setminus \mathcal{L}_\ell$.

For a fixed $\ell \in \{1,\ldots,2n-2\}$, let $e \in \mathcal{R}_{3\ell}$. From the analysis above we know that $2e^T f(K_p,K_v,B^Tq)\big|_{q=e+\mu^{-1}(K_p,K_v,r)} < 0$ for all $e \in \mathcal{R}_2$. In particular, the derivative is strictly negative for all e in the set $\{e \in S\ :\ e_i = 0\,, \forall i \in \mathcal{L}_\ell\,, e_i \geq 0\,, \forall i \in \bar{\mathcal{L}}_\ell\}$. Since $\dot{V}(e)$ is a continuous function of its arguments, there must exist a sufficiently small value $\bar{e}_\ell > 0$ such that $\dot{V}(e)$ continues to be strictly negative on the subset $\mathcal{D}_{3\ell} = \{e \in \mathcal{R}_{3\ell}\ :\ e_i > -\bar{e}_\ell\ \forall i \in \mathcal{L}_\ell\ ,\ e_i \geq 0\ \forall i \in \bar{\mathcal{L}}_\ell\}$. Now, consider the remaining portion of $\mathcal{R}_{3\ell}$, namely the set of points $\mathcal{R}_{3\ell} \setminus \mathcal{D}_{3\ell}$ where $e_i \leq -\bar{e}_\ell$ for all $i \in \mathcal{L}_\ell$. Choose $0 > -\eta_i \geq \max\{\epsilon_i(-\gamma', K_{vi}, r_i), \epsilon_i(-\bar{e}_\ell, K_{vi}, r_i)\}$. Since $\epsilon_i(e_i, K_{vi}, r_i)$ is a monotonically increasing function of e_i which is zero at zero, for all $e_i \leq -\bar{e}_\ell$, we have $e_i\Psi_{ti}(\epsilon_i(e_i,K_{vi},r_i)) = N_i e_i \leq -N_i\bar{e}_\ell < 0$. Let $N_{i\ell} > 0$ be such that $M - \sum_{i\in\mathcal{L}_\ell} N_{i\ell}\bar{e}_\ell < 0$. Then for all $N_i \geq N_{i\ell}$, for all $e \in \mathcal{R}_{3\ell} \setminus \mathcal{D}_{3\ell}$, $\dot{V}(e) < 0$. This is true for all $\ell \in \{1,\ldots,2n-2\}$, hence for each $i = 1,\ldots,n$ there exists N_{i3} such that $N_i \geq N_{i3}$ guarantees that $\dot{V}(e) < 0$ on $\mathcal{R}_3$.
The thesis is inferred by letting, for $i = 1,\ldots,n$, $N_i^* = \max\{N_{i1}, N_{i3}\}$ and $N_i \geq N_i^*$. ∎

V. Numerical results

Proposition 2 provides a solution to the pressure regulation problem using binary controllers. A similar result also holds for quantized controllers. We illustrate this presenting some numerical results regarding the use of quantized controllers for the example system shown in Fig. 2. In the simulation the pressures dp_4 and dp_5 are (practically) controlled to a reference value of 0.5 [bar]. The parameters of the quantized controllers introduced in Section IV.A are the following: $\delta = 0.5$, $j = 3$, and $u_0 = 4$. The numerical results are shown in Fig. 7. From the figure, it is seen that the performance is as expected. In fact, only 5 different controller levels are necessary to obtain control performance comparable with the one obtained with the continuous P-controller proposed in [2].

VI. Conclusion

We have presented a class of quantized controllers distributed over a (hydraulic) network. The main motivation for the investigation is the control of large-scale district heating systems in the presence of communication constraints.
The quantized controllers we have investigated guarantee semi-global practical regulation of the end-user pressure drops for a fairly general class of hydraulic networks.
The next steps consist of investigating the robustness of these controllers to phenomena such as delays and other deleterious effects.

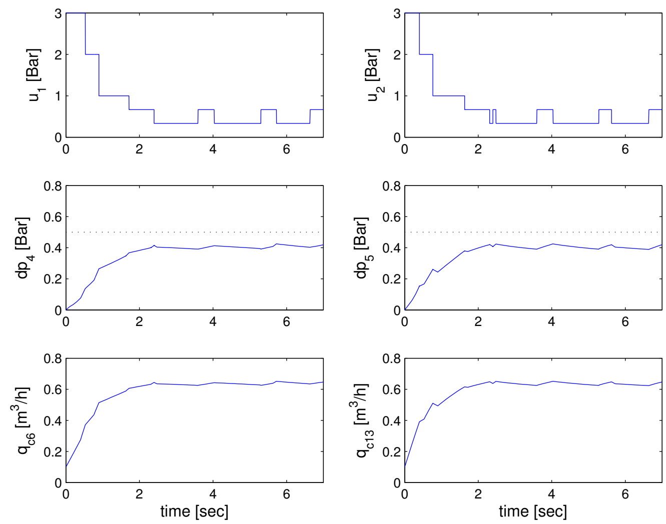

Fig. 7. The control inputs u_1, u_2, the controlled variables dp_4, dp_5, and the flow through valves c_6, c_{13} obtained with the proposed quantized controller.

Moreover, we shall study networks and protocols which allow the actual implementation of the proposed controllers in a networked environment.

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